REPORT No. 26

THE VARIATION OF YAWING MOMENT DUE TO ROLLING

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PREVIOUS WORK

The aerodynamical constants of an airplane necessary for the discussion of stability are partly observed and partly calculated. Among the calculated coefficients is N_p, which is the variation of yawing moment due to rolling. In the Technical Report of the Advisory Committee for Aeronautics (London), 1912–13, Reports and Memoranda No. 77, being an Investigation into the Stability of an Aeroplane, etc., by L. Bairstow and others, on page 157 it is stated that "for the wings it will be seen that whilst L_p is proportional to the slope of the lift curve, N_p is proportional to the slope of the drag curve. Hence N_p will be one-tenth of L_p at angles slightly above that giving maximum lift/drag, and may become zero, or even slightly negative, in a machine flying at about the angle of minimum drag. The effect of the rudder and body will be appreciable in most machines. N_p will be variable between the limits 0 and 40."

In a contribution on "Dynamical stability of aeroplanes," by Jerome C. Hunsaker, Smithsonian Miscellaneous Collections (Washington), vol. 62, No. 5, June, 1916, pages 55–57, the calculation of N_p is carried out in detail along the lines laid down in the above quotation and values for the machine under discussion (a design by Capt. V. E. Clark, United States Army) run from 0 to 57.

An essential argument in these derivations of a value for N_p is that the roll produces an increased line of attack on an elementary area of the right wing and a diminished angle of attack on the corresponding element of the left wing; and that, consequently, the variation of the yawing moment should be calculated from the slope of the drag curve. Now, as a matter of fact, this would be certainly correct in case the wing were a flat plane to which the resultant pressure remained always normal, independently of the angle of attack; but it is by no means certain that the argument is valid in the case of cambered wings, where a change in the angle of attack produces a change in the direction of the resultant force as well as a change in its magnitude. It would seem to be more likely to be correct to take account of the change in direction of the resultant force, as is the case in the calculation of X_w, the variation in the X force due to vertical velocity. In the case of the flat plane X_w is a negative quantity, owing to the diminution of the resultant pressure (its direction remaining invariant) when the angle of attack is diminished; but the change in direction of the resultant force on the cambered wing is so much more important than the diminution in its magnitude (owing to the large value of the lift relative to the drag) that the value of X_w for the airplane becomes actually positive instead of negative.

A NEW CALCULATION

It would seem that in calculating N_p the change in the X force (not in drag alone) should be used. It would be possible to make the calculation on this basis in the following manner:

Let dy be an element of length along the wing, and S its span. If m is the mass of the airplane in slugs, $mX_w w$ is the variation in the actual X force, due to w; and for the element dy

the proportional part would be $mX_w wdy/S$. The moment of this change is the change itself multiplied by its arm y, viz., $mX_w wydy/S$; and the total value of the variation in the moment

$$mN_p p = \int_{-S/2}^{+S/2} mX_w wydy/S.$$

The value of w is negative, and equal to $-yp$. On substituting this value, we have

$$N_p = \int_{-S/2}^{+S/2} -X_w y^2 dy/S = -X_w S^2/12.$$

When the values for the Clark design are substituted, in which $S=40$, the following results are found:

U =	−112.5	−78.2	−65.3	−54
i =	0°	3°	6°	12°
X_w =	+ .356	+ .249	+ .245	0
N_p =	− 47.5	−33.2	−32.7	0

whereas, according to Hunsaker, the values of N_p are

N_p =	0		+33.5	+57.0

It will be observed that N_p by this calculation is negative, instead of positive, and that the numerical values are large for small angles of attack, small for large angles. Not only the sign of N_p is changed, but the general trend of numerical values is reversed.

EFFECT ON STABILITY

Fortunately the value of N_p is not of very serious moment in the discussion of stability. The expression in which it is most important is the approximate form of the damping in the type of motion which Hunsaker (loc. cit., p. 71) calls the Dutch roll, and which corresponds to the quadratic factor of the biquadratic that governs the lateral motion, namely:

$$D^2 + \left(\frac{C_2}{B_2} - \frac{E_2}{D_2}\right)D + \frac{B_2 D_2}{B_2^2 - A_2 C_2} = 0\cdot$$

$$\frac{C_2}{B_2} - \frac{E_2}{D_2} = \frac{L_r}{K_C^2}\left(\frac{N_p}{L_p} - \frac{N_v}{L_v}\right).$$

The value of L_p is negative. That of N_v is also negative, and those of L_v and L_r are positive. If, therefore, N_p be positive, as found by Bairstow and Hunsaker, the expression in the parenthesis is the difference of two quantities; and in order to insure stability, it is necessary that this difference be positive—that is, we must have N_v/L_v greater numerically than N_p/L_p. Now, N_v and L_v occur in the expression which determines spiral stability or instability, and the ratio $-N_v/L_v$ is desired small for spiral stability, whereas it is desired large for stability in the Dutch roll. Thus, there arises the necessity for a very fine compromise in the relative magnitudes of N_v, L_v, N_p, L_p, in order that the machine may be stable both spirally and in the Dutch roll.

If, however, the value of N_p be negative, as is indicated by my calculation above, both terms in the parenthesis $(N_p/L_p - N_v/L_v)$ are positive for most attitudes of flight which have been examined, and the machine is stable in the Dutch roll without the necessity for any fine adjustment as compared with the spiral case. This should be a matter of some relief to the conscientious designer critical of the dynamic stability of his design.

CHECK ON THE CALCULATION

In so far as my argument for the calculation of N_p is just, a similar argument could be given for obtaining a calculated value of L_p in terms of Z_w, with the result:

$$L_p = Z_w S^2/12.$$

The values of L_p as calculated by this formula would be:

U =	-112.5	-78.2	-65.3	-54.0
i =	$0°$	$3°$	$6°$	$12°$
Z_w =	-5.62	-3.77	-2.92	-1.0
L_p =	-749	-493	-389	-133

whereas the values given by Hunsaker are

L_p =	-631		-319	-224

The values of L_p are obtained rather easily by measuring the damping of the model when oscillating about the X axis, and, consequently, the measured values should be fairly trustworthy. The only two cases in the table in which a comparison can fairly be made are those in the first and last columns corresponding to highest and lowest speeds; for the integer in the third column is not an experimental value, but one obtained by interpolation. At the highest speed the calculated value of L_p is nearly 20 per cent too high; whereas at the lowest speed it is distinctly too low.

It would not be surprising if a calculated value based on Z_w should be too high, for the experimental method of determining Z_w is to compare the Z forces for the model when set at different angles of pitch relative to the fixed direction of the air current. Now, it is a common observation in our wind tunnel at the Massachusetts Institute of Technology that when the orientation of the model relative to the wind is quickly changed, a very considerable time elapses before the forces reach their steady value. It appears as though it took a reasonable amount of time for the stream lines in the fluid to change from one steady direction to another.

If this be so, it would be impossible for the stream lines to accommodate themselves to the oscillatory motion in the experimental determination of L_p as fully as they accommodate themselves to the changed orientation in the experimental determination of Z_w. The result would be that the effective value of Z_w, which should be used in the calculation of L_p, might be a considerable amount below the measured value of Z_w. No such explanation could be given for the discrepancy between the values as calculated and observed of L_p at the lowest speed, for the direction of the change is reversed. There is, however, the possibility that the value of Z_w as calculated from the experiments should be considerably in error, because this value must be obtained either by an interpolation in a table of values, or by estimating the slope of an experimentally determined curve, and either of these processes is one in which it is difficult to obtain accuracy, because the experimental errors or an error of judgment in fairing a curve are extremely effective in vitiating the value obtained for the rate of change of the ordinate in the vicinity of any particular point on the curve.

A comparison may be made for the Curtiss J. N. 2., from Hunsaker's data (loc. cit., p. 78) and these reports, First Annual Report, 1915, pp. 47–49:

U	i	Z_w	L_p(obs.)	L_p(calc.)	X_w	N_p(new)	N_p(old)
-115.5	$1°$	-3.95	-314	-427	$.162$	-17.2	0
-63.8	$15°.5$	$-.673$	-78	-73	$-.292$	$+31.5$	$+37.7$

Here the calculated L_p at low speed checks very well with that observed, but is again considerably too high at high speeds. The reversal of sign of X_w for this machine has brought the two values of N_p for low speed near together.

EXPERIMENTAL CHECK

In the British report (1912–13), Reports and Memoranda No. 78, being the Experimental Determination of Rotary Coefficients, by L. Bairstow, etc., on pages 177–179 there is outlined an experimental method of measuring the value of L_r, the variation of the rolling moment due to yawing by a somewhat intricate experimental procedure, based on the theory of forced oscillations. In a similar manner, the variation of yawing moment due to rolling, N_p, could be determined. It seems, however, that the measurement of the amplitude of the forced oscillation would determine only the numerical magnitude, and not the sign of L_r or N_p as the case might be; and that for the experimental determination of the sign it will be necessary to observe the phase difference between the forced oscillation and the periodic applied force. This should not be a difficult thing to observe, but it is quite possible that if one knew, or thought he knew, in advance what the sign of the result should be, he might overlook the matter of checking the sign by an observation on the phase difference of the two motions.

It is probable that before any great dependence can be put upon the calculated value of N_p or similar aerodynamic coefficients, an extended comparison of calculated values with experimentally determined ones will be necessary; and I have not offered the above discussion so much for the purpose of attempting a definitive determination of the value of N_p as for the purpose of finding out the possibility of making a calculation which would seem to be just as reasonable if not more reasonable than those before given; and which does, as a matter of fact, lead to a value of N_p of negative sign instead of one with a positive sign. The important thing for the discussion of stability is not so much the numerical value of N_p, unless N_p be positive, as the assurance that N_p is negative, if, indeed, it be negative.—(*Extract from lectures given at the Massachusetts Institute of Technology to a special course in aeronautical engineering for Army and Navy officers, May to September, 1918.*)

NOTE.—In a book on Aeronautics, by Cowley and Levy, which has just come to hand, there is found on page 261 a table in which the value of N_p (in a notation different from that of Bairstow and Hunsaker) is negative; but the details of the calculation which lead to the value are not given, so that it can not be determined whether or not the negative value is intended or is a typographical error.